

Live Soap: Order, Fluctuations and Instabilities in Active Smectics

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We construct a hydrodynamic theory of noisy, apolar active smectics, in bulk suspension or on a substrate. Our predictions include: quasi-long-ranged smectic order in dimension $d = 2$, and long-ranged in $d = 3$, extending previously published results to all dynamical regimes; Kosterlitz-Thouless melting to an active nematic at high *and* low concentrations in $d = 2$; nonzero second-sound speed parallel to the layers; the suppression of giant number fluctuations by smectic elasticity; instability to spontaneous undulation and flow in bulk contractile smectics; a layer spacing instability, possibly oscillatory, for large enough extensile active stresses.

Active particles [1] in a state of orientational order [2, 3] exhibit fluctuations [4–7] and flow properties [8–13] that differ strikingly from those in equilibrium systems with the same spatial symmetry. Translationally ordered active matter has received less attention [10, 14]. This paper studies active systems with spontaneously broken translation-invariance in one direction – active smectics – in a broad range of dynamical regimes.

We consider apolar systems of particles carrying an axis of orientation disposed on average along the normal to the smectic layers (i.e., Smectics A [15]), with active stresses [11] pulling in or pushing out, i.e., contractile or extensile, along that axis. A variety of such models with this symmetry are possible, depending on conservation laws and geometry; we study five of these. The simplest is systems with *no* conserved quantities. Such a model describes the dynamics of Rayleigh-Bénard roll patterns [16, 17], and spontaneously layered flocks of self-propelled apolar entities, reproducing or dying while in motion [18], on a substrate which serves as a momentum sink. The second is layered flocks moving on a substrate *with* number conservation, but without momentum conservation. The third is *bulk* layered systems in a background fluid with momentum conservation treated in the “Stokesian”, i.e., viscosity-dominated, limit appropriate for colloidal or microbial active systems, and the fourth is such systems *confined* between no-slip walls, where the surfaces are a momentum sink and the hydrodynamic interaction is screened at long wavelengths. We conclude by analyzing bulk systems in a background fluid at wavelengths beyond the Stokesian regime, where inertia dominates over viscosity.

Our results: (i) In all the five cases we study, smectic order, when dynamically stable, is long-ranged in the presence of noise in dimension $d = 3$ and quasi-long-ranged in $d = 2$. This reinforces and extends the findings of [14]. (ii) The active smectic undergoes a transition to an active nematic as the concentration of active particles is varied, in all five cases. In two dimensions, “reentrance” [19] *necessarily* occurs: the active nematic occurs at both large and small concentration, with the active

smectic at intermediate concentrations. Both transitions, in the “no-conservation” case, are of Kosterlitz-Thouless type [20]; the nature of the transitions for the other four models is unknown. (iii) Bulk smectic liquid crystals in the Stokesian limit are hydrodynamically *stable* to the presence of *extensile* active stresses with magnitude below a threshold value [21] (iv) Active smectics, unlike their orientationally ordered counterparts [2–4, 6], have finite concentration fluctuations, whose magnitude, however, diverges, as activity approaches the threshold. (v) Bulk active smectics with contractile active stresses are generically unstable without threshold to spontaneous undulations [22] and flow. (vi) Confined active smectic suspensions are in general stable at long wavelengths for small enough active stresses of either sign. Beyond a threshold value of activity the confined system too undergoes an instability, which is likely to be oscillatory for the extensile case. (vii) For stable bulk active smectics, the speed of the smectic second sound mode is nonzero for propagation parallel to the smectic layers. Our findings apply to active smectics in a wide range of settings including vibrated granular layers [23] and the Rayleigh-Bénard problem [16]. Agitated 2DEGs [24], where Coulomb and magnetic-field effects enter, will be discussed elsewhere[25].

We begin with the simplest case: active elements whose number is *not* conserved, spontaneously condensed into a uni-directional periodic structure, i.e., a smectic A, with mean layer normal $\hat{\mathbf{n}}$ along $\hat{\mathbf{z}}$. The only hydrodynamic field in this case is the broken symmetry variable u giving displacements of the layers along $\hat{\mathbf{z}}$. This model also describes Rayleigh-Bénard stripes [16], where the modulated field is the local temperature, which is not a conserved quantity. This case was dealt with briefly in [14]. The hydrodynamic, long-wavelength model for the dynamics of the u -field, retaining terms permitted by symmetry, including $u \rightarrow -u$, $z \rightarrow -z$ [26], to leading order in gradients and in powers of u , reads

$$\partial_t u = \tilde{B} \partial_z^2 u + D \nabla_{\perp}^2 u - \tilde{K} \nabla_{\perp}^4 u + f^u, \quad (1)$$

where f^u is a Gaussian, zero-mean spatiotemporally

white noise with variance 2Δ . The term with coefficient D [27] is forbidden by rotation-invariance of the free energy in an *equilibrium* smectic without an aligning field. It is however permitted here simply because rotation-invariance *at the level of the equation of motion*, which is all one can demand in an active system, does not rule it out. Its physical content is that layer curvature produces a local vectorial asymmetry which must cause directed motion of the layers as this is a driven system. Symmetry does not fix the sign of D . An undulation instability [28] arises if $D < 0$. For positive D the linearity and spatial homogeneity of (1) makes it straightforward to show via spatial Fourier transform that the variance $\langle |u(\mathbf{q}, t)|^2 \rangle = \Delta / (\tilde{B}q_z^2 + Dq_\perp^2)$ in Fourier space for small wavevectors $\mathbf{q} = (\mathbf{q}_\perp, q_z)$. This implies [14] a real-space variance $\langle u^2 \rangle = \int_{\mathbf{q}} \langle |u(\mathbf{q}, t)|^2 \rangle$ that is finite in $d = 3$, corresponding to long-range smectic order; in $d = 2$, $\langle u^2 \rangle \sim \log L$ for system size L , corresponding to quasi-long-range order. This establishes result (i) for the simplest case.

Ignoring the \tilde{K} term, which is irrelevant at long distances, rescaling $\mathbf{r}_\perp = \mathbf{r}'_\perp (D/\tilde{B})^{1/2}$, $z' = z$, and expressing u in terms of the angle $\theta(\mathbf{r}') \equiv (2\pi/a)u(\mathbf{r})$ where a is the layer spacing, we can rewrite (1) in the form

$$\partial_t \theta = \tilde{B} \nabla'^2 \theta + f^\theta, \quad (2)$$

with rescaled noise statistics

$$\langle f^\theta(\mathbf{r}', t) f^\theta(\mathbf{0}, 0) \rangle = \left(\frac{2\pi}{a} \right)^2 \Delta \left(\frac{\tilde{B}}{D} \right)^{\frac{d-1}{2}} \delta(\mathbf{r}'_\perp) \delta(z') \delta(t). \quad (3)$$

Defining $\kappa \equiv \tilde{B}^{(3-d)/2} D^{(d-1)/2} a^2 / (2\pi^2 \Delta)$ it can be shown that the steady-state probability distribution for θ implied by (2), (3), is $\exp[(-\kappa/2) \int d^d r' (\nabla' \theta)^2]$, identical to that for a thermal equilibrium XY model with a stiffness/temperature ratio κ .

This equivalence to an equilibrium XY model implies that topological defects (i.e., dislocations) in an active smectic in dimension $d = 2$ unbind, driving the system into the active nematic phase, [20] when $\kappa = 2/\pi$, i.e., when $2\pi^2 \Delta / a^2 (\tilde{B} D)^{1/2} = \pi/2$. This locus is plotted in the Δ - D plane in figure 1(a).

Since it is a purely active effect, we expect $D \propto c_0$. We also expect the noise strength Δ to get an active contribution proportional to c_0 , and a c_0 -independent thermal contribution proportional to $k_B T$. Hence, varying c_0 maps out a straight line with positive intercept on the Δ -axis in the Δ - D plane, as illustrated in figure 1(a). As is clear from that figure, this experimental locus can only enter the active smectic region by crossing the active smectic to active nematic phase boundary twice. Hence our conclusion that re-entrance is inevitable in two dimensions for these systems.

In three dimensions, the situation is quite different, because *equilibrium* smectics are *stable* against dislocations

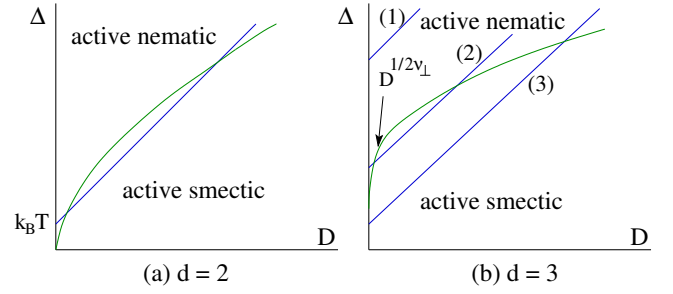


FIG. 1. Phase diagram of active smectics, in (a) $d = 2$ and (b) $d = 3$.

in $d = 3$, since dislocations are line defects that remain bound even in the absence of the D term in (1) [29]. As a result, the active smectic- active nematic phase boundary does *not* go all the way down to $\Delta = 0$ at $D = 0$ in $d = 3$. It *does*, however, develop an infinite downward slope at $D = 0$, as can be deduced by the following argument: for small D , as the transition is approached, the system will act like an equilibrium ($D = 0$) system until the D term in (1) becomes comparable to the \tilde{K} term at in-plane wave vectors $q_\perp \sim \xi_\perp^{-1}$, where ξ_\perp is the equilibrium, in-plane correlation length for smectic order. This condition leads to $D/\xi_\perp^2 \sim \tilde{K}/\xi_\perp^4$, which implies

$$\xi_\perp \sim \sqrt{\tilde{K}/D}. \quad (4)$$

Near the equilibrium AN transition, $\xi_\perp \propto |T - T_{AN}|^{-\nu_\perp}$ where ν_\perp is the “thermodynamic” equilibrium correlation length exponent in the \perp direction; see reference [30] for a further discussion. Since Δ plays the role of temperature here, (4) leads to a shift $\propto D^{1/2\nu_\perp}$ in the critical Δ_{cr} . Both theory [30] and experiment [31] find $1/2\nu_\perp < 1$, so the phase boundary in figure 1(b) has infinite slope as $D \rightarrow 0$.

The locus in the Δ - D plane mapped out by varying c_0 remains a straight line; now, however, re-entrance, though still obviously a possibility (e.g., for locus 2 in figure 1(b)), can be avoided, as on locus 3 in figure 1(b).

As in $d = 2$, in $d = 3$ the active smectic to active nematic transition is in the XY model universality class for the model with no conservation laws. While the other four dynamical models we study in this paper are *not* equivalent to *any* equilibrium XY model, they have exactly the same scalings of their equal time u - u correlations as this simplest model. Hence, we expect similar phase diagrams, with the critical $\Delta_{cr} \propto D^{1/2}$ for small D in $d = 2$. However, the universality class of the phase transition in these other models may be different.

We now turn to our second model, in which the number of active particles *is* conserved, but momentum is not. Now the concentration c of active particles with mean c_0 and $\delta c \equiv c - c_0$, joins the broken symmetry variable u as a hydrodynamic field. For *apolar* phases, the equations of motion must be unchanged if $u \rightarrow -u$ and $z \rightarrow -z$

simultaneously. The c -dependent term in the u equation of motion that is lowest order in spatial derivatives that respects this symmetry is $\propto \partial_z c$; hence, the equation of motion for u is:

$$\partial_t u = \tilde{B} \partial_z^2 u + D \nabla_\perp^2 u - \tilde{K} \nabla_\perp^4 u + \tilde{C} \partial_z \delta c + f^u, \quad (5)$$

with f^u having the same statistics as those in (1).

The equation of motion for the concentration c must, due to conservation of total number of particles, be expressible in the form: $\partial_t c = -\nabla \cdot \mathbf{J}^c$. Gradient expanding the current subject to the symmetry constraints gives:

$$\mathbf{J}^c = -\hat{\mathbf{z}}[(A_z - W^c) \partial_z \delta c + W^c c_0 \nabla_\perp^2 u + C_{zz} \partial_z^2 u] - \nabla_\perp [A_\perp \delta c + (C_{\perp z} + W^c c_0) \partial_z u] + \mathbf{f}^c \quad (6)$$

where the Gaussian noise $\mathbf{f}^c = (f_\perp^c, f_z^c)$ has statistics

$$\langle f_i^c(\mathbf{r}, t) f_j^c(\mathbf{0}, 0) \rangle = (\Delta_\perp^c \delta_{ij}^\perp + \Delta_z^c \delta_{ij}^z) \delta^d(\mathbf{r}) \delta(t). \quad (7)$$

In (6) we have included an active current [5] $W^c \nabla \cdot (\mathbf{c} \mathbf{n} \mathbf{n})$ with $\hat{\mathbf{n}} = (\hat{\mathbf{z}} - \nabla u)/|\hat{\mathbf{z}} - \nabla u|$, where W^c is a phenomenological coefficient. In an *equilibrium* two component smectic, (6) and (7) would hold, but with the constraints $W^c = 0$ and $C_{\perp z}/C_{zz} = A_\perp/A_z = \Delta_\perp^c/\Delta_z^c$.

The two key results that emerge from (5)-(7) are that, for $q \rightarrow 0$ and for all directions of \mathbf{q} , the equal-time correlators $\langle |u_{\mathbf{q}}|^2 \rangle \propto 1/q^2$ for *all* directions of \mathbf{q} , and that $\langle |\delta c_{\mathbf{q}}|^2 \rangle$ is finite. As before, this q^{-2} scaling of u fluctuations implies translational order is quasi-long-ranged in $d = 2$, and long-ranged in $d = 3$. The finite concentration fluctuations, which result from the smectic elasticity \tilde{B} , imply the absence of giant number fluctuations (our result (iv)).

We next consider active smectics suspended in a fluid medium. The total momentum of suspended particles and ambient fluid is conserved; the corresponding momentum density \mathbf{g} is therefore slow and hydrodynamic. The other hydrodynamic fields u and c remain as well, of course. We'll assume overall incompressibility, so that total (particle + fluid) mass density $\rho = \rho_0 = \text{constant}$ and $\nabla \cdot \mathbf{v} = 0$, where $\mathbf{v} \equiv \mathbf{g}/\rho$ is the velocity field.

Conservation of total momentum reads $\partial_t \mathbf{g} = -\nabla \cdot \boldsymbol{\sigma}$, with a linearized stress tensor

$$\boldsymbol{\sigma} = p \mathbf{I} - \boldsymbol{\eta} (\nabla \mathbf{v} + \nabla \mathbf{v}^T) + \boldsymbol{\sigma}^{(el)} + \boldsymbol{\sigma}^a + \boldsymbol{\sigma}^N, \quad (8)$$

with p the fluid pressure, $\boldsymbol{\eta}$ the viscosity tensor, and the elastic force density $-\nabla \cdot \boldsymbol{\sigma}^{(el)} = -\mathbf{n} \delta F / \delta u$, with

$$F = \frac{1}{2} \int d^d x [B(\partial_z u)^2 + K(\nabla_\perp^2 u)^2 + A(\delta c)^2 + 2C \delta c \partial_z u]. \quad (9)$$

Here B and K are layer compression and bend moduli respectively, A the osmotic modulus, and C a cross-coupling. The active stress [3, 8] $\boldsymbol{\sigma}^a = -W \mathbf{c} \mathbf{n} \mathbf{n}$, with negative and positive W corresponding respectively to extensile and contractile stresses, while $\boldsymbol{\sigma}^N$ is noise. That

$\boldsymbol{\sigma}^a \propto c$ follows because W is the activity *per particle*. As in any smectic A, the layer normal \mathbf{n} is geometrically locked to the displacement field: $\delta \mathbf{n} \equiv \mathbf{n} - \hat{\mathbf{z}} \simeq -\nabla_\perp u$.

The resulting equation of motion for \mathbf{v} , linearized in \mathbf{v} , u and $\delta c = c - c_0$, with c_0 the mean concentration, reads

$$\rho_0 \partial_t \mathbf{v} = -\nabla p + \hat{\mathbf{z}} [B \partial_z^2 u - K \nabla_\perp^4 u + (C + W) \partial_z \delta c] - W c_0 (\hat{\mathbf{z}} \nabla_\perp^2 u + \partial_z \nabla_\perp u) - \boldsymbol{\Gamma} \cdot \mathbf{v} + \mathbf{f}^v, \quad (10)$$

where $-\boldsymbol{\Gamma} \cdot \mathbf{v} = \nabla \cdot (\boldsymbol{\eta} \nabla \mathbf{v})$, $\mathbf{f}^v = \nabla \cdot \boldsymbol{\sigma}^N$ is a momentum-conserving noise, and $\langle \sigma_{ij}^N(\mathbf{0}, 0) \sigma_{kl}^N(\mathbf{r}, t) \rangle = 2 \Delta_{ijkl} \delta(\mathbf{r}) \delta(t)$. Δ_{ijkl} is uniaxial, symmetric in ij and kl and under interchange of ij with kl , and thus has five independent components in $d = 3$. In thermal equilibrium $\Delta_{ijkl} \propto \eta_{ijkl}$, but not in general nonequilibrium systems [32]. We take $\Delta_{ijkl} = \Delta_v (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})$ and $\eta_{ijkl} = \eta \delta_{jk} \delta_{il}$. The linearized hydrodynamic equation of motion for u is

$$\partial_t u = v_z + \tilde{B} \partial_z^2 u + D \nabla_\perp^2 u - \tilde{K} \nabla_\perp^4 u + \tilde{C} \partial_z \delta c + f^u; \quad (11)$$

where the noise f^u has statistics as in (1) [33].

The equation of motion for c is: $\partial_t c = -\nabla \cdot \mathbf{J}^c$ with \mathbf{J}^c given by equation (6).

For simplicity, we take $\Delta_\perp^c = \Delta_z^c \equiv \Delta^c$, $C_{\perp z} = C \Delta^c = C_{zz}$, and $A_\perp = A \Delta^c = A_z$, where A and C are as in (9), and define $C \Delta^c \equiv E$, $A \Delta^c \equiv G$. Activity now enters only through the W terms in (10).

To study the Stokesian limit, we neglect inertia and acceleration, impose incompressibility $\nabla \cdot \mathbf{v} = 0$ in (10), and solve for \mathbf{v} in terms of u and c . Inserting the result in (11), and defining $\Phi \equiv -\partial_z u$, we find that the spatial Fourier transforms $\delta c_{\mathbf{q}}$, $\Phi_{\mathbf{q}}$ obey

$$\partial_t \Phi_{\mathbf{q}} = -M_{\mathbf{q}} \{ [B q_z^2 + W c_0 (q_z^2 - q_\perp^2) + K q_\perp^4] \Phi_{\mathbf{q}} - (C + W) q_z^2 \delta c_{\mathbf{q}} \} + [\partial_t \Phi_{\mathbf{q}}]_P - i q_z (f_{\mathbf{q}}^u + M_{\mathbf{q}} f_{z\mathbf{q}}^v) \quad (12)$$

$$\partial_t \delta c_{\mathbf{q}} = (E q^2 + 2W c_0 q_\perp^2) \Phi_{\mathbf{q}} - (G q^2 - W c_0 q_z^2) \delta c_{\mathbf{q}} - i \mathbf{q} \cdot \mathbf{f}_{\mathbf{q}}^c \quad (13)$$

where $M_{\mathbf{q}} \equiv q_\perp^2 / \eta q^4$, and $[\partial_t \Phi_{\mathbf{q}}]_P$ summarizes the ‘‘permeative’’ $\tilde{B}, \tilde{K}, \tilde{C}, D$ terms from (11), which are of higher order in wavenumber than those shown explicitly in (12).

Suppose $B > CE/D = C^2/A$, so that when activity $W = 0$ the smectic state is stable. Let $|W| > C > 0$; a similar analysis holds for $C < 0$. At small q , where $[\partial_t \Phi_{\mathbf{q}}]_P$ is negligible, it is clear from (12) that negative (i.e., extensile) W , can lead to an instability with \mathbf{q} along z , i.e., a modulation in layer spacing. However, the layer compression modulus B always stabilizes this when $(B - |W|c_0) > 0$. Thus, the system is stable for small enough $|W|$, establishing our result (iii).

For contractile active stresses $W > 0$, we see from (12) and (13) that the most unstable modes have \mathbf{q} in the \perp direction, in which neither the layer compression elasticity nor the coupling to the concentration act. Hence,

the instability threshold for $W \rightarrow 0$ in the limit of large system size, as in [8, 9]. The instability causes splay and self-generated flow, as in active nematics [8, 9]. For smectics, this is a spontaneous version of the Helfrich-Hurault [15, 28] undulation instability.

We turn next to the effects of confinement. Consider an active smectic with layers normal to the z direction, confined between no-slip walls parallel to the xz plane, a distance ℓ apart. We start with (10) but with both \mathbf{f}^v and $\mathbf{\Gamma} \sim \eta/\ell^2 \equiv \Gamma$ nonzero at zero wavenumber because the walls are a momentum sink. Solving the modified (10) for the (now) fast variable \mathbf{v} in terms of the slow u and c and inserting the result in (11) yields

$$\partial_t \Phi_{\mathbf{q}} = -(\bar{B}q_z^2 + \bar{D}q_x^2 + \bar{K}q_x^4)\Phi_{\mathbf{q}} + \bar{C}q_z^2 \delta c_{\mathbf{q}} - iq_z(f_{\mathbf{q}}^u + M_{\mathbf{q}} f_{z\mathbf{q}}^v), \quad (14)$$

while equation (13) continues to hold. Here $\bar{B} = \tilde{B} + (B + Wc_0)M_{\mathbf{q}}$, $\bar{D} = D - Wc_0M_{\mathbf{q}}$, $\bar{K} = \tilde{K} + KM_{\mathbf{q}}$, and $\bar{C} = \tilde{C} + (C + W)M_{\mathbf{q}}$. The wave vector \mathbf{q} now lies in the xz plane and the mobility $M_{\mathbf{q}} = (q_x^2/q^2)(1/\Gamma)$, in contrast to the bulk system, does not diverge at small q , so that terms involving $M_{\mathbf{q}}$, which arise from the (screened) hydrodynamic interaction, are of the same order in wavenumber as the permeative terms $[\partial_t \Phi_{\mathbf{q}}]_P$. Ignoring the concentration field, we see that the relaxation rate of layer displacements with wavevector in the x direction is now proportional to $[D - (c_0W/\Gamma)q_x^2/q^2]q_x^2$ for $q_x \gg q_z$ and $\{\tilde{B} + [(B + c_0W)/\Gamma]q_x^2/q^2\}q_z^2$ for $q_z \gg q_x$. Thus, there is a range of parameters for which the active smectic is stable (result (vi)). For other parameter ranges instabilities occur; e.g., if $D = 0$ in (11), an undulation instability occurs for $\mathbf{q} \perp \mathbf{z}$, despite confinement.

The instability that arises in the extensile ($W < 0$) case when $|W| > B$ is interesting. Equations (14) and (13) have the same form as the linear part of the Fitzhugh-Nagumo [34, 35] equation, which exhibits sustained oscillations under rather general conditions. We speculate that such oscillations could also occur here; i.e., a breathing smectic. We will explore this in future work.

We now turn to fluctuations in the bulk Stokesian limit. For $q_z \rightarrow 0$, δc drops out of (12), and it is then easy to check for extensile activity that $\langle |u_{\mathbf{q}}|^2 \rangle = 2\Delta_v/|W|c_0q_{\perp}^2$. Somewhat more tedious algebra, which we'll present elsewhere, shows that $\langle |u_{\mathbf{q}}|^2 \rangle \sim 1/q^2$ for all directions of \mathbf{q} . We can also see from (12) that, in the Stokesian regime, time correlations of u decay at a nonzero rate in all directions for $q \rightarrow 0$. These facts imply that, in bulk active smectic suspensions as well, the coupling to u via the E term in (13) won't lead to diverging concentration fluctuations in general. $\langle |\delta c_{\mathbf{q}}|^2 \rangle$ does diverge, however, upon approaching the extensile instability, with a correlation length $\sim (B - |W|c_0)^{-1/2}$.

We conclude with second sound. Leaving the steady Stokesian regime, taking acceleration ($\partial_t \mathbf{v}$) into account, and working at long wavelengths where viscosity is neg-

ligible, makes the coupled dynamics of \mathbf{v} and u , for wavevectors in the plane of the layers $\rho_0 \partial_t \mathbf{v} = -\nabla p - Wc_0 \hat{\mathbf{z}} \nabla_{\perp}^2 u$; $\partial_t u = v_z$; $\nabla \cdot \mathbf{v} = 0$, can readily be seen to give sound waves with a speed $\sqrt{-Wc_0/\rho_0}$.

In conclusion, we have constructed the dynamical equations for active smectics, both in bulk suspensions and in confined systems in contact with a momentum sink. Our theory is generic, applicable to any driven system with spontaneous stripe order. We show, extending [14], that noisy active smectic order is long-ranged in dimension $d = 3$ and quasi-long-ranged in $d = 2$ for all dynamical regimes, and that active smectic suspensions have a nonzero second sound speed parallel to the layers. For $d = 2$ we predict a Kosterlitz-Thouless transition from active nematic to active smectic, with a re-entrant nematic at low concentration. We show that smectic elasticity suppresses the giant number fluctuations and extensile instabilities that occur in active nematics, but that bulk contractile systems exhibit an active undulation instability. Active extensile stresses, if strong enough, give rise to a "breathing" instability which is likely to be oscillatory. Our results should apply to a wide range of active systems, including horizontal layers of granular matter agitated vertically or fluids heated from below.

We look forward to detailed experimental tests of our predictions.

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